NBIM DISCUSSION NOTE

On risk premium variation

18 March 2011

This section provides a brief introduction to modern financial economics and theories of discount factor variation

Main findings

- During the last four decades, financial research has moved from striving to understand how new information about future payoffs is incorporated into market prices to striving to account for how and why the discount factor of these payoffs varies over time and across assets. This has profound practical implications. Multiple dimensions of risk with time-varying discount factors potentially open up a more demanding portfolio theory.

- The average investor must by necessity hold the market portfolio, but the market portfolio is not necessarily optimal for all investors. Thorough understanding of how own liabilities, own non-traded risks, and careful identification of comparative advantages and disadvantages relative to other investors, might give reasons to pursue dynamic portfolio strategies that differ from those of both the marginal and the average investor.

- These strategies might include departures from mean-variance trade-offs, such as engaging in trading strategies which aim at exploiting known premiums in cross-section and time series, e.g. employing value-weighted rebalancing rules.

- The analytical framework used by modern financial economics is also shared with modern applied fiscal theory, which, among other things, gives a transparent rationale for the Norwegian government's fiscal spending rule ("handlingsreglen").

The central tenet of modern finance

The central tenet of modern finance is that prices are equal to expected discounted payoffs, formally

$$p_t^i = E_t\left(m_{t,t+1}x_{t+1}^i\right),$$

where E_t denotes the expectation conditional on information available at time t, $m_{t,t+1}$ is a (stochastic) discount factor, and x_{t+1}^i is a random payoff of a traded asset i. m_{t+1} is stochastic or random since it is not known with certainty at time t. Payoff x_{t+1}^i might, for example, be the sum of the next period's dividend and price: $x_{t+1}^i = d_{t+1}^i + p_{t+1}^i$. In a world where the discount factor for asset i does not vary over time, the pricing formula can be expressed via the standard present value formula, $p_t^i = E_t(x_{t+1})/R^i$, where R^i represents a constant discount factor.

Using the definitions of covariance and the real risk-free rate at time t, the expected risk premium (the expected return in excess of the risk-free rate) for asset i can be written as follows (see appendix for derivation):

$$p_t^i = \frac{E_t(x_{t+1})}{R^i}$$

The more negative the covariance, the poorer the insurance properties of the asset, the lower and more "discounted" the price, and the higher the expected return. For newcomers to asset pricing theory, this might be surprising. The variance of the return R_t^i is per se irrelevant and does not measure risk or the risk premium. Only the covariance of the return with the stochastic discount factor $m_{t,t+1}$ matters for the expected risk premium.

Efficiency and asset pricing "anomalies" and "puzzles"

Historically, financial research focused on the expectations operator $E_t(\cdot)$ and strived to understand how market prices incorporate information as it becomes available. The introductory textbook view of asset pricing and portfolio theory is simple. There is one source of systematic risk, the market index. Investors understand this and choose a desired portfolio consisting of a combination of an approximately risk-free interest-bearing asset and the market index. Within this analytical framework, active management means uncovering inefficiently priced assets, or "chasing alpha".

The research agenda associated with this simple, stylised portfolio model was to "test market efficiency". Researchers found that new information is quickly incorporated into market prices. Tentative conclusions, such as that "chasing alpha" might, on average, be an unprofitable activity, are partially based on these findings.

Despite new information about future payoffs being efficiently and almost immediately processed by markets, studies show that expected returns vary more over time and across assets than can be accounted for by traditional models where the variation of discount rates is constant. In these models, changes in future expected payoffs are calibrated to match the statistical properties of observed time series. These observations gave rise to a large number of apparent "puzzles" and "anomalies", such as the "term spread puzzle" and the "credit spread puzzle".

One set of empirical regularities in financial markets is that returns on stocks, long-term government bonds and corporate bonds have, on average, been higher than returns on short-term government bonds. The "equity premium puzzle" describes the failure of models with constant variation of the discount factor to account for the observed excess return of equities over short-term government bonds.

Cross-sectional factors in stocks, such as value, small-cap, momentum, accruals and issuance, are well-documented, as is the way that low current stock valuations relative to fundamentals (for example, dividend yield, earnings growth and price/earnings to growth) tend to be followed by high subsequent returns. Another well-documented set of empirical regularities is that returns on certain long-short strategies have had a predictable component: the returns on currency carry trades are predictable based on interest rate differentials. Various strategies involving a definition of "liquidity provision" have some of the same properties. Analyses of hedge fund strategies have highlighted other possibilities, including index puts and other option strategies, which seem to offer predictable premiums.

Discount factor dynamics – a unifying theme of modern financial economics

Based on persuasive empirical documentation of time-varying expected returns, the focus for modern financial research has therefore moved to further documenting how, and accounting for why, the stochastic discount factor m varies as much as it does over time. This is one of the unifying themes of modern asset pricing theory. In the introduction to a recent paper, John Cochrane (December 2010) summarises:

> Prices should equal expected discounted value. In 1970, Gene Fama argued that *the expected part*, "testing market efficiency," provided the framework for organizing finance research... Finance research today is really all about *the discounted part*: *How* risk premia vary over time and across assets, *why* they do so, and *how* to apply this understanding. "Efficiency" isn't wrong; it just doesn't describe the focus of what we do. When we see information, it is quickly incorporated into market prices. When we see anomalies – and we see many – informational frictions aren't an interesting alternative. Anomalous discount rate variation is.

A starting point for any portfolio strategy that includes deviations from the market portfolio must be to account for how these seemingly "puzzling" empirical regularities can be sustained over time. If any mass of investors systematically try to exploit these empirical regularities, the regularities will vanish. To believe that these factors have "behavioural" explanations and are caused by "irrationality" seems an untenable position.

Asset pricing and the real economy

In some way, the discount factor between the two periods t and $t+1$ (m_{t+1}) must be related to the marginal investor and to the ratio of the marginal utility of wealth between these two consecutive periods

$$m_{t,t+1} = \frac{V_W(t+1)}{V_W(t)}.$$

Loosely speaking, the marginal value of wealth answers the question "how much better off would you be if I give you one additional unit of wealth (one krone, say)?" The discount factor is high at times and in states where investors strongly want more wealth – and would be willing to give up a lot of wealth at other dates or in other states to get it.

Understanding the dynamics of the marginal value of wealth is intimately linked to modern dynamic macroeconomics. The centrepieces of macroeconomics are the equation of marginal rates of substitution to marginal rates of transformation (loosely speaking, that the marginal time value of wealth is equal to the marginal product of capital), and the allocation of consumption and investment across time and states of nature. The same mechanisms that ensure that these equilibrium conditions hold are the same mechanisms that ensure market clearing in asset markets.

Formally, this ratio is the first-order condition of the hypothetical marginal investor who has maximised the net present value of current and future welfare V

$$V(W_t, z_t) = E_t \sum_{i=0}^{\infty} \theta^i u(c_{t+i})$$

given a period-by-period budget constraint and an optimal, time-consistent decision rule (or policy function) h

$$c_t = h(W_t, z_t)$$

where W_t is wealth at time t, z_t is other state variables at time t, is a time-discount factor, c is whatever is the source of welfare ("consumption"), and the function $u(\cdot)$ is the transformation of the source of welfare ("consumption") into a measure of welfare.[1]

The Norwegian government's fiscal spending rule (*handlingsreglen*) can be accounted for as the decision rule or policy function resulting from a similar, simple, transparent optimisation problem. It is just one of very many examples of how modern asset pricing and portfolio theory provide the same analytical tools and ways to frame the questions as modern dynamic macroeconomics, including analysis of long-run fiscal policy.

In the remainder of this section, we will very briefly survey some scientific contributions that have attempted to account for time and cross-sectional variation in discount factors. These contributions fall into at least three broad categories: (i) model economies that explicitly take into account non-tradable risks, (ii) model economies that explicitly model market frictions, and (iii) model economies where markets are complete and frictionless, but where structural parameters, such as preference parameters, are calibrated such that they capture underlying incompletenesses and frictions. Finally, we will suggest one possible framework for thinking systematically about the potential dynamic implications of having preferences that differ from those of the marginal investor in a framework where discount factor volatility matches that of the data.

Model economies with incomplete markets and non-tradable risk

In order to account for cross-sectional variation and time-varying volatility of market discount factors, researchers have worked with models that explore the connection between asset market valuations and investors' risks associated with their non-market, non-tradable wealth. The central object of investigation is the effect of market incompleteness and non-tradable risk on market pricing. For individuals, these risks might include the net present value of the return on human capital; for corporations, the net present value of business capital; and for countries, the net present value of future output or tax revenues.

These models can also be used as analytical frameworks to identify how and why an individual investor's effective risk preferences, and hence also portfolio composition, differ from those of the implicit marginal investor.

Fama and French's (1996) suggested explanation for the value premium belongs to this category. For investors whose value of future human capital is highly correlated with the fortunes of value firms, investing in these firms provides poor insurance. These investors would therefore require a higher

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1   The mathematical representation of the net present value of future welfare for the hypothetical marginal investor is similar to the expression of the price of an asset as a function of future cash flow. If payoff x_{t+1}^i is the sum of the next period's cash flow and price, $x_{t+1}^i = d_{t+1}^i + p_{t+1}^i$, substituting forward and using the law of iterated expectations, the price can be expressed as the expected sum of discounted cash flows

$$p_t^i = E_t \sum_{\tau=1}^{\infty} m_{t,t+\tau} d_{t+\tau}^i$$

where $m_{t,t+\tau} = \prod_{j=1}^{\tau} m_{t+j-1,t+j}$

expected return for investing in these companies than in companies whose fortunes are less correlated with the value of their human capital. If there are enough investors in this category, that might be sufficient to account for the observed excess return on value firms.

The key premise of this line of research is that traded assets do not span the space of risk that investors care about. One implication of these contributions is that assets that are highly correlated with non-traded risk tend to have higher excess returns (be discounted by a higher factor) than assets that are less correlated with non-traded risk. Another implication is that all investors do not share the same portfolio. Instead investors will adapt their portfolios to hedge their non-tradable outside risk.

On average, the financial wealth of individuals is smaller than other non-tradable wealth components, such as the net present value of future labour income. To maximise the risk-adjusted net present value of total wealth, individuals could be expected to use their financial wealth partially to offset specific risks associated with their non-tradable wealth components. The "equity home bias puzzle", defined by, among others, French and Poterba (1991) and Tesar and Werner (1995), is a statement about the apparent failure of individuals to do this; instead of hedging country-specific risk by shorting assets of the home country and countries that are highly correlated to it, and going long in assets from countries that are less correlated with the home country, individuals in most countries seem to have a sub-optimal home bias in their financial assets.

Model economies with frictions

Another strategy to account for time and cross-sectional variation in discount factors, and to identify own comparative advantages, has been to explicitly identify and test frictions that might contribute to the observed time and cross-sectional variation. One possible categorisation of models with explicit frictions is: 1) segmented markets, 2) institutional finance or intermediated markets, and 3) liquidity premiums.

In a segmented market, some investors participate in some markets, and other investors participate in other markets. This means that the basic first-order condition

$$p_t^i = E_t\left(m_{t+1}^j x_{t+1}^i\right),$$

holds only for investors j who are matched to security i. Risks are then shared only between investors in a specific group, not across groups. This feature limits risk-bearing activity and therefore leads to the emergence of premia that are not related to aggregate risks. Thus, one basic consequence of "segmented markets" is that risks are not shared across market participants as they are in the standard model.

One example might be how a general borrowing constraint affects different age groups differently. Constantinides, Donaldson and Mehra (2002) argue that the equity premium can partially be put down to the fact that young households are borrowing-constrained and so equity risk is not shared among age groups.

The literature on "intermediated markets" or "institutional finance" applies to a different, vertical, separation of investor from payoff. Investors use delegated managers to handle their assets. Then, frictions or principal/agent problems in the delegated management relationship spill over into market prices for the assets.

A long tradition in asset pricing recognises that some assets have higher or lower discount rates in compensation for greater or lesser "liquidity". Defining "liquidity", modelling it and understanding it deeply are still, however, open questions. Recently, both theoretical and empirical work have emphasised liquidity as a systemic factor, not just an individual-security characteristic. Times when all assets become illiquid are high marginal utility events, so assets that pay off well in times of aggregate illiquidity should offer lower expected returns.

Frictionless, complete-markets model economies

When modelling markets as frictionless, we gain transparency and tractability. We can then exploit consumer/investor first-order conditions to tie the discount factor to changes in the marginal utility of wealth. Since Mehra and Prescott (1985), Hansen and Jagannathan (1991) and others showed that the standard model with additive separable preferences could not account for observed risk premia (like the equity premium), many approaches have been undertaken to provide preference-based (or production-based) theories consistent with observed asset pricing facts. Surveys of the literature can, for example, be found in Cochrane (2001), and a list of "exotic preferences" generated by the ensuing research can be found in Backus, Routledge and Zin (2004).

Preferences play several important roles in financial economics. They provide, in principle, an unchanging feature of the model in which decision-makers can be confronted with a wide range of different asset classes, environments and institutions.

While there are many routes, those seeking to "reverse-engineer" preferences from key asset pricing facts have mainly followed two approaches. The first involves habit formation or "catching up with the Joneses" (see, for example, Campbell and Cochrane 1999). The second approach seeks to relax the classic assumption that welfare can be separated and added up across time and over states of nature (see Epstein and Zin 1989).

In dynamic, stochastic settings, individuals have preferences over time, preferences towards risk, and combinations of the two. Partially due to analytical convenience, the most common preference specification in macroeconomics and financial economics has been the additive expected structure

$$V = \sum_{i=t}^{\infty} \theta^{i-t} \sum_{\forall s} \pi(s^t) u(c_{t,s})$$

where θ is the time-discount factor, $\pi(s^t)$ is the probability of history s^t, and u is a period/state welfare function.

Additive separability is, however, a very strong and restrictive assumption for preferences over time and states of nature. In addition, for reasonable parameters, the associated stochastic discount factor is almost constant. As proven by, among others, the many "puzzles" and "anomalies" surveyed earlier in this section, its mean, standard deviation and time-variation of volatility are such that it cannot account for the basic asset pricing facts.

Building on work by, among others, Kreps and Porteus (1978) and Johnsen and Donaldson (1985), Epstein and Zin (1989) derived a recursive preference specification that satisfies the fundamental axioms for choice, is analytically tractable, and is not restricted by the assumption of additive separability. Epstein and Zin (1991) pioneered employing utility specifications that are non-separable across time and across states in the asset pricing literature. It has since gained a dominant status and is often referred to as the standard preference specification of modern asset pricing.

Epstein and Zin propose a recursive formulation of utility that abandons the strong assumption of additive separability across time and states of the world. In their specification, the time aggregator is

$$U_t = \left[(1 - \beta) c_t^\rho + \beta \mu (U_{t+1})^\rho \right]^{\frac{1}{\rho}}$$

where $\mu(\cdot)$ is the risk aggregator function, often also referred to as the certainty-equivalent function. $\mu(U_{t+1})$ is a future welfare from the next period and onwards, measured in terms of current welfare (or consumption). The time aggregator returns the weighted sum of current welfare and welfare from the next period and onwards. The lower ρ is, the more individuals would prefer current welfare and the certainty equivalent of future welfare to move together.

The long-run welfare risk aggregator $\mu(U_{t+1})$, or certainty-equivalent function, is a similar functional form.

The smaller (more negative) the parameter α is, the more individuals dislike variation in future welfare

$$\mu(U_{t+1}) = [E_t(U_{t+1}{}^{\alpha})]^{\frac{1}{\alpha}}$$

Without going into details in this section, the difference between the parameters $\alpha - \rho$ is a measure of individual preference for early resolution of uncertainty. Very loosely speaking, it is a matter of how impatient individuals are to learn what the future will look like.

If $\rho = \alpha$, the specification reduces to a model where utility is additively separable across time and states. Models with non-time-separable utilities (habits, durables) also distinguish risk aversion and intertemporal substitution, but not in such a simple way.

If $\rho \neq \alpha$, we see a second term: expected returns will depend on covariances with changes in the utility index, capturing news about the investor's future prospects, as well as on covariances with consumption growth.

The associated stochastic discount factor is

$$m_{t+1} = \beta \left(\frac{c_{t+1}}{c_t}\right)^{\rho-1} \left(\frac{U_{t+1}}{\mu(U_{t+1})}\right)^{\alpha-\rho}$$

Work by Bansal and Yaron (2004), Hansen, Heaton and Li (2008) and many others indicates that many of the asset price "anomalies" can be accounted for in model economies where uncertainty is time-varying and decision-makers have this class of recursive preferences. In calibrations with this set of preferences, most of the time-varying volatility in the discount factor comes from the second part of the expression, $\left(\frac{U_{t+1}}{\mu(U_{t+1})}\right)^{\alpha-\rho}$, which is the ratio of future welfare (broadly defined) to the certainty-equivalent of future welfare. In other words, fluctuations in the long-run growth prospects of the economy and time-varying level of uncertainty associated with long-run growth (consumption or output volatility) drive changes in the stochastic discount factor, in other words the market price of risk.

This preference specification in environments with time-varying uncertainty has also been employed to address a rich array of other asset market issues. A few examples: Kiku (2006) can account for the shortcomings of the capital asset pricing model (CAPM) and consumption CAPM (C-CAPM) in accounting for the cross-sectional differences in mean returns. Eraker (2006) and Piazzesi and Schneider (2005) consider the implications of the marginal investor's sensitivity to growth-rate risks for the risk premia on US Treasury bonds and can account for some of the average premium puzzles in the term structure literature. Chen, Collin-Dufresne and Goldstein (2009) and Chen (2010) show that they can use this framework to account for the credit spread and leverage puzzles of the corporate sector.

Deviations from standard additive-separable preferences seem also to be necessary to analyse how "fat tails" and other deviations from normality are priced.[2] Backus, Chernov and Martin (2009) use information about the volatility of higher-order moments implied by equity index options to derive information about deviations from normality and implications for market preference parameters.

Heterogeneity in sensitivity to long-run risk and higher-order moments (such as "fat tails") is also a natural point of departure for portfolio heterogeneity, both on average in cross section and dynamically over time (rebalancing). As stated earlier in this section, in frictionless environments, preference heterogeneity is not necessarily literally "preference" heterogeneity. It might also be a reduced-form representation of other forms of heterogeneity, such as differences on the liability side, different regulations or differences in how investors are affected by the same regulations.

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2   Deviations from normality are moments like skewness and excess kurtosis. Martin (2010) extends the Epstein-Zin asset pricing model to incorporate information about the higher-order moments – equivalently cumulants – of consumption growth. He argues that the importance of higher-order moments might be a double-edged sword: parameters that govern higher-order moments (or cumulants) are critically important for asset pricing, but extremely hard to calibrate.

Backus, Routledge and Zin (2009) have made some progress in analysing risk-sharing and asset allocations with heterogeneous recursive preferences. Their working paper shows how the dynamics of output growth not only reflect standard mean-variance tradeoffs, but also tradeoffs involving the timing of the resolution of uncertainty. This framework would also be very suitable for analysing the dynamic portfolio allocation of an investor with different risk sensitivities to those implied by the "marginal investor". In particular, this framework is suited to studying dynamic reallocation as uncertainty (and the market price of risk) changes.

Concluding remarks

In recent decades, research into asset pricing has increasingly focused on documenting and accounting for cross-sectional and time variation in discount factors. This change has been prompted by persistent empirical regularities: the well-known "puzzles" and "anomalies" can almost all be traced back to discount factor variation rather than problems of information incorporation.

Academic research is just about to understand the implications of the view of the world that there are multiple dimensions of risk with shifting premiums. This might have a profound impact on strategies for the management of large, long-term funds such as the Government Pension Fund Global.

Even though it is clear that investors with different risk sensitivities and different exposures to non-tradable risks should hold different portfolios, these insights should be applied with caution. The complement of any asset management strategy that aims at systematically exploiting discount factor variation must be a convincing theory for why it is optimal for the average investor to hold the market portfolio, given preferences, risks and price dynamics. For example, a statement about own comparatively higher-than-average risk-bearing capabilities must be coupled with convincing arguments for why the majority of other investors have lower risk-bearing capabilities.

A large and growing body of academic literature has documented how several of the observed premiums (including the credit and term premiums) can be accounted for by exposure to changes in uncertainty associated with long-run growth prospects. The source of what is represented by differences in risk preferences might be differences in non-tradable risk or how the investor is affected by market frictions. This should not only be postulated, but also documented. A starting point for gaining further understanding of the existing rebalancing rule, as well as formulating new strategies to systematically exploit time-varying market discount factors, may be to study dynamic portfolio problems for investors with risk preferences that differ from those of the implicit marginal investor.

Deviations from the market portfolio might not only be based on differences in regulation and risk-bearing capabilities. This is also by itself a as well as reason to deviate from the market portfolio. Modern macroeconomics has an analytical framework that is similar to and complements that of modern asset pricing and portfolio theory. Investment strategies for the Government Pension Fund Global could potentially be structured in order to partially hedge the non-tradable macroeconomic risk facing the owners of the fund.

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Appendix

We have

$$p_t^i = E_t(m_{t+1})E_t(x_{t+1}^i)$$

Using the definition of covariance

$$p_t^i = E_t(m_{t+1})E_t(x_{t+1}^i) + cov_t(m_{t+1}, x_{t+1}^i),$$

gross returns are defined as the ratio of payoffs to price, $R_t^i = \frac{x_{t+1}^i}{p_t^i}$, hence

$$E_t\left(m_{t+1}\frac{x_{t+1}^i}{p_t^i}\right) = E_t(m_{t+1}R_t^i) = 1.$$

Using the definition of covariance and the real risk-free rate at time t, $R_t^f = \frac{1}{E_t(m_{t+1})}$

$$\frac{E_t(R_t^i)}{R_t^f} + cov_t(m_{t+1}, R_t^i) = 1.$$

If expected excess returns are defined as $E_t(R_t^{ei}) \equiv E_t(R_t^i) - R_t^f$, then

$$E_t(R_t^{ei}) = -R_t^f cov_t(m_{t+1}, R_t^i).$$

The expected risk premium or "expected excess return" is determined by the covariance with the discount factor.