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Alternatives to a Market-value-weighted Index

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We study alternative portfolio construction methods in an attempt to improve the return-to-risk characteristics of market value weights. To understand the investability of these approaches we introduce a novel way to measure the investment capacity of a portfolio relative to the market-value-weighted index.

Main findings

- Empirical research has shown that portfolio weights based on market capitalisation yield mean-variance inefficient portfolios. We study how the return-to-risk characteristics of a market-value-weighted equity portfolio can be improved by applying alternative portfolio construction methods. We consider approaches based on heuristics and optimisation. We find that the outperformance of approaches based on heuristics and optimisation is partly driven by known factors, such as the value premium and the low-risk anomaly.

- Approaches categorised as heuristic are typically related to known risk factors, because these approaches utilise characteristics known to predict stock returns. Fundamental weights (FW) overweight cheap (value) stocks and underweight expensive (growth) stocks. GDP weights (GDPW) are similar in spirit to fundamental weights at country level. Equal weights (EW) tilt towards small firms as well as value stocks. Equal-risk-budget (ERB) strategy equalises volatility exposure and thus benefit from the low-risk anomaly.

- Optimisation-based approaches, such as equal risk contribution (ERC), most-diversified portfolio (MDP) and minimum-variance (MV), have proven to provide more efficient portfolios than a market-value-weighted portfolio.

- If a large investor were to allocate money to an alternative portfolio, it is important to know how much one can deploy capital to a specific portfolio. We introduce a novel measure of relative investment capacity (RIC) which answers precisely how much capital one can deploy to an alternative approach relative to the market-value-weighted portfolio.

- At individual stock level FW has the highest RIC. FW and ERB have high RICs at industry portfolios. When looking at country allocation FW stands out as the highest RIC. GDP weights at a country level have rather disappointing RICs. The rest of the approaches considered have very low RICs. Low RICs tend to go hand in hand with high turnover and require more active management to rebalance the portfolio.

- Alternative approaches help to improve portfolio return-to-risk characteristics but this comes at a cost of lower investment capacity and higher turnover. These findings should be of interest and relevant to large investors considering alternative portfolio weights either as a policy benchmark or as an investment strategy.

Introduction

The Capital Asset Pricing Model (CAPM) is one of the most celebrated asset-pricing models, of which the cornerstone is the mean-variance framework of Harry Markowitz.[1] Selecting the most desirable portfolio in a mean-variance framework implies that all investors want to hold the portfolio with the highest Sharpe ratio, i.e. the market-value-weighted portfolio. A vast body of empirical research shows that this theoretically appealing prediction does not hold in the real world, implying that the market-value-weighted portfolio is not mean-variance-efficient.[2]

In the CAPM universe, all investors know precisely and agree upon expected returns, volatilities and correlations. Merton (1980) shows that mean-variance-optimal portfolio weights are especially sensitive to the expected returns. This means that a very small tweak in expected return inputs can result in very different portfolio weights. This may explain why many alternative weighting approaches agnostic to expected returns provide more efficient return-to-risk characteristics than the market-value-weighted portfolio.

Ideally one would improve on market value weights either by purely reducing risk or by purely enhancing returns.[3] In practice, approaches are embedded with both attributes, making it hard to disentangle the specific benefits of the effort. For example, approaches based on optimisation require the variance-covariance matrix to be invertible, which can be attained via clustering with respect to risk factors. It is then very difficult to say whether the optimisation approach adds value or whether the real value added comes from bearing some risk not captured by the market-value-weighted portfolio.

We study alternative portfolio construction methods in an attempt to improve the return-to-risk characteristics of market value weights. Consistent with Chow, Hsu, Kaleshnik and Little (2011), we analyse rule-based weighting methods and optimisation separately. The heuristic weighting methods that are considered in this note are equal weights (EW), equal risk budget (ERB), fundamental weights (FW) and GDP weights (GDPW). Similarly, we take a closer look at equal risk contribution (ERC), most diversified portfolio (MDP) and minimum variance (MV) which are based on optimisation. These alternatives to market value weights provide appealing return-to-risk characteristics. Our empirical results show that the improvement in return-to-volatility profile is related to the value and low-risk factors.

To understand the investability of these approaches we introduce a novel way to measure the investment capacity of a portfolio relative to the market-value-weighted index. If one were to use an alternative index as a benchmark, desirable properties of this index would be to have a sufficient and stable investment capacity. We introduce a novel measure of relative investment capacity (RIC) to help understand how much capital one can deploy to an alternative approach relative to the market-value-weighted portfolio. The RIC measure shows that fundamentally-weighted and equal risk budget portfolios have reasonably high relative investment capacities while all other approaches have low investment capacity. Minimum-variance portfolios have particularly low relative capacity and GDW weighted country portfolios have a disappointingly low RIC. We advocate RIC to be used as a measure of investment capacity when considering alternative indices either as an investment strategy or as a policy benchmark. RIC should be especially relevant for large investors.

We proceed as follows. First, we discuss the relevant literature and describe alternative weighting approaches to portfolio construction. We then introduce a novel measure of relative investment capacity before we move on to discuss the data and research methodology. In the following section, we analyse the exposure to different risk factors and discuss the relative investment capacity of all alternative approaches considered. Final section concludes.

1 See Markowitz (1952) and Sharpe (1964).

2 See, for example, Fama and Macbeth (1973) and Gibbons, Ross and Shanken (1989).

3 Chow, Hsu, Kaleshnik and Little (2011) classify strategies into two categories: (1) heuristic-based weighting methodologies and (2) optimisation-based weighting methodologies.

Review of portfolio construction approaches

Traditional equity market indices have been based on market-value-weighted portfolios – perhaps because, in theory, the CAPM suggests that the market portfolio is mean-variance-efficient and should be the optimal way to make investment decisions. Gibbons, Ross and Shanken (1989) and Grinold (1992) use the Gibbons-Ross-Shanken (GRS) test and find that indices based on market capitalisation are not mean-variance-efficient, inconsistent with the CAPM. Haugen and Baker (1991) compare the Wilshire 5000 index to a low-volatility portfolio and present empirical results showing the return-to-risk characteristics of the market-value-weighted index are inferior to the low-volatility portfolio. Clarke, de Silva and Thorley (2006) confirm the same finding using a longer and more recent dataset from 1968 to 2005. The results are also consistent with Ang, Hodrick, Xing and Zhang (2006), who conclude that stocks with higher historical idiosyncratic volatility have lower realised returns. In general, the paper concludes that realised standard deviation is lowered by about a quarter, and market beta is reduced by about a third, compared to the market-value-weighted benchmark. This paper also shows that minimum-variance portfolios tend to have a bias toward both a value and a small-firm effect. Even after imposing ex-ante neutrality constraints, they find that some of the value added of minimum-variance portfolios can be attributed to the value factor.

The market-value-weighted portfolio may be prone to assign a relatively high weight to winners and a low weight to losers. This would be the opposite of value investing where one buys undervalued assets and sells overvalued assets. This particular feature would lead to underperformance of the market-value-weighted index, as past long-term winners are likely to be overvalued and become subsequent losers.

Because of this inefficiency, numerous statistical and econometric approaches have been developed to construct more efficient portfolios. We review these portfolio construction approaches by categorising them into heuristic and optimisation approaches. Table 1 summarises and compares these approaches.

Table 1: Comparison of portfolio construction approaches

	Logic	Performance	Note
Market Value Weighted (MVW)	Use market capitalisations as portfolio weights.	Underperformance due to negative value tilt via concentration in overvalued large stocks.	Methodology is widely used and transparent and has high investment capacity. Products are easily available. Concentrated weight in overpriced assets drags performance.
Fundamentally Weighted (FW)	Use non-market-value measures of size as portfolio weights.	Overperformance due to positive value tilt via avoiding overvalued large stocks.	This approach outperforms the market-value-weighted index, has a fairly large capacity, is transparent and is easy to build, although the capacity is slightly lower and turnover is higher relative to the market cap weights.
Equally Weighted (EW)	Use equal weights (1/N) to build a portfolio.	Overperformance due to positive value tilt via avoiding overvalued large stocks.	Very easy to build and transparent approach. This approach lacks a theoretical foundation and tilts the portfolio towards illiquid small caps.
Equal Risk Budget (ERB)	Weighting based on equal risk budgets (excluding correlation).	Overperformance due to low-volatility anomaly and business cycle component.	Fairly easy to build and transparent approach. This approach is related to MDP and MV. Also known as risk parity.
Equal Risk Contribution (ERC)	Weighting based on marginal risk contribution (including correlation).	Overperformance due to low-volatility anomaly and business cycle component.	Needs optimisation tool to build. This approach is related to ERB, MDP and MV.
Most Diversified Portfolio (MDP)	Portfolio that maximises the diversification ratio.	Overperformance due to low-volatility anomaly and business cycle component.	MDP is linked to ERB. They produce the same portfolios if the correlations are equal across assets.
Minimum Variance (MV)	Find MV portfolio using optimisation.	Overperformance due to low-volatility anomaly.	This portfolio has low risk but may have relatively low return. Related to RP and MDP.

Market value weights (MVW)

Market value weights are motivated by the CAPM where the market portfolio is efficient and should be the most desirable portfolio (under the simplified theoretical assumptions). MVW assigns the percentage portfolio weights according to the market capitalisation of assets:

$$Market\ Value\ Weight: w_i = \frac{m_i}{\sum m_j}$$

Here w_i is the portfolio weight of company *i*. m_i is the market value of company *i* and $\sum m_j$ is the sum of market values of all companies in the portfolio.

In practice, the market-value-weighted portfolio has a large weight of assets whose market price is high and a low weight of relatively cheaper assets. If long-term winners are, on average, overpriced and long-term losers are underpriced, then the MVW portfolio is tilted towards overvalued assets generating low subsequent returns when prices correct to a normal level. This particular phenomenon is utilised by value investors.

Fundamental weights (FW)
Arnott, Hsu and Moore (2005) advocate an idea that weighting the assets in a portfolio according to a fundamental metric rather than the market value of the assets provides a better return-to-risk profile. FW portfolios thus are more efficient than market-value-weighted portfolios. A fundamentally-weighted index typically calculates portfolio weights using accounting measures of size or economic fundamentals rather than market values:

$$Fundamental\ Weight: w_i = \frac{b_i}{\sum b_j}$$

Here w_i is the portfolio weight of company *i*. b_i is the book value of company *i* and $\sum b_j$ is the sum of the book values of all companies in the portfolio. The accounting measure could, in principle, be any accounting-based size variable which has low correlation with the market capitalisation of the company. Examples of accounting measures used are book value of equity, number of employees, average five-year cash flow, revenue, gross sales and dividends. This approach gives a lower weight to expensive companies and a higher weight to cheaper companies. FW tilts the portfolio towards value stocks and provides a higher expected return than a market-value-weighted portfolio.

GDP weights (GDPW)
GDP weights use the gross domestic product of countries as a measure of economic size and are thus similar in spirit to fundamentally-weighted portfolios. However, this approach can be used only to weight countries in global indices and not individual stocks within countries. In this approach, one weights countries according to their GDP relative to global GDP:

$$GDP\ Weight: w_k = \frac{gdp_k}{\sum gdp_j}$$

Here w_k is the portfolio weight of country *k*. gdp_k is the gross domestic product of country *k* and $\sum gdp_j$ is the total gross domestic product of all countries in the portfolio.

Equal weights (EW)
Equally-weighted portfolios have been widely used in academic literature. Depending on the investment opportunity set, an EW portfolio may provide dramatically varying weights and provide arbitrary risk exposures over time. Chow, Hsu, Kalesnik and Little (2010) discuss how an equally-weighted Russell 1000 portfolio is much more volatile and has a high small-cap exposure relative to an equally-weighted S&P 500 portfolio. EW has high tracking error and also high turnover relative to market cap weighting.

The EW portfolio assigns a naïve 1/N weight to each asset as follows:

$$Equal\,Weight\!: w_i = \frac{1}{N}$$

Here w_i is the weight for stock *i* and *N* is the number of stocks in the portfolio.

DeMiguel, Garlappi and Uppal (2009) carefully study 14 different optimistion models against an equally-weighted portfolio (EW) approach using 8 different datasets.[4] Interestingly they find that EW, the ultimate shrinkage estimator, outperforms all optimisation models in terms of out-of-sample Sharpe ratio.

Equal risk budget (ERB)

An equal risk budget (ERB) portfolio assigns equal volatility risk budgets to all assets in the portfolio.[5] ERB is a risk-weighted version of the EW portfolio providing a more balanced risk profile than EW. ERB uses volatility as a risk measure and ignores correlation risk. ERB is based on pre-determined and targeted risk budgets and is not based on optimisation. The formula below shows the weights for ERB:

$$Equal\ Risk\ Budget\!: w_i = \frac{1/\sigma_i}{\sum 1/\sigma_j}$$

Here w_i is the portfolio weight of stock *i* and σ_i is the volatility of stock *i*. $\sum 1/\sigma_j$ is the sum of inverse volatilities in the portfolio. For example, suppose an asset A has a volatility half of the volatility of another asset B. An investor following ERB would invest twice as much money in asset A as in asset B. This example shows how an investor following the ERB approach equates volatility risk exposure across assets, whereas the dollar amount allocated to each asset will differ according to the risk budget.

Since ERB assigns a higher weight to low-risk assets, this approach is related to the low-beta anomaly, which is the observation that low-beta stocks have positive alpha whereas high-beta stocks have negative alpha. Frazzini and Pedersen (2010) argue that this anomaly is driven by the fact that many investors are not able to use leverage, and instead of taking risk via leverage they allocate directly to high risk, for example high-beta stocks.

Equal risk contribution (ERC)

In an equal-risk-contribution (ERC) portfolio, risk contribution is equalised by considering variance and correlation, which is discussed in Maillard, Roncalli and Teiletche (2010). ERC is an approach where the risk contribution of each asset is distributed equally. Unlike in ERB, which considers only the volatility risk, ERC takes the covariance of assets into consideration. In ERC one equalises each asset's risk contribution by using the following formula from Maillard, Roncalli and Teiletche (2010):

$$Marginal\ Risk\ Contribution\!: \frac{\partial\sigma(w)}{\partial w_i} = \frac{w_i\sigma_i^2 + \sum_{i\neq j} w_i\sigma_{ij}}{\sigma(w)}$$

4 The 14 models are: sample-based mean-variance, Baysian diffuse prior, Bayes-Stein, Bayesian Data-and-Model, minimum variance, value-weighted market portfolio, MacKinlay and Pastor missing-factor model, sample-based mean variance with short sale constraints, Bayes-Stein with short-sale constraints, minimum variance with short-sale constraints, minimum variance with generalised constraints, Kan and Zhou three-fund model and Garlappi, Uppal and Wang multi-prior model. The eight datasets are: ten sector portfolios of the S&P 500 and the US equity market prrtfolio, ten industry portfolios and the US equity market portfolio, eight country indexes and the world index, SMB and HML portfolios and the US equity market portfolio, 20 size- and book-to-market portfolios and the US equity market portfolio, 20 size- and book-to-market portfolios and the MKT, SMB and HML portfolios, 20 size- and book-to-market portfolios and theMKT, SMB, HML and UMD portfolios, simulated data.

5 Asness, Frazzini and Pedersen (2011) use ERB labelled as risk parity in a multi-asset framework.

Maillard, Roncalli and Teiletche (2010) show theoretically that the ERC portfolio falls somewhere in between the minimum-variance (MV) and EW portfolios, and also that

$$\sigma_{MV} \leq \sigma_{ERC} \leq \sigma_{1/n}$$

Most diversified portfolio (MDP)

The most-diversified portfolio is a concept advocated by Choueifaty and Coignard (2008) and Choueifaty, Froidure and Reynier (2011) which is based on maximising a diversification ratio. The numerator of the diversification ratio is a weighted average of volatility risk budgets and the denominator is the portfolio volatility as shown by the following formula:

$$Diversification\ Ratio = \frac{Weighted\ Average\ Volatility}{Portfolio\ Volatility} = \frac{\sum w_i \sigma_i}{\sqrt{\sum\sum w_i w_j \sigma_i \sigma_j \rho_{ij}}}$$

Minimum variance (MV)

MV is a well-known special case in the mean-variance framework. If past returns do not predict future returns, the MV portfolio could be a good starting point for a practical application, as argued by Chopra and Ziemba (1993). To build an MV portfolio, one minimises portfolio variance as follows:

$$Portfolio\ Variance\ = \sum\sum w_i w_j \sigma_i \sigma_j \rho_{ij}$$

The MV portfolio allocates a higher weight to low-risk assets and thus achieves lower risk than the market-value-weighted portfolio. The MV portfolio has historically had a higher average return than the market-value-weighted portfolio, which is puzzling because risk reduction should not necessarily yield higher expected returns. Clarke, de Silva and Thorley (2010) claim that 80 to 90 percent of long-only minimum-variance portfolio risk is systematic, and thus that the surprisingly strong performance of the minimum-variance portfolios is related to the well-known CAPM critique, starting with Black, Jensen and Scholes (1972), that low-beta stocks have high returns indistinguishable from high-beta stocks.

Relative investment capacity

Investment capacity is of paramount importance for a large investor making investment decisions. Understanding to what extent one can deploy capital into a given portfolio helps an investor to decide to what extent this portfolio is investable. Low investment capacity and high turnover would limit the scale at which one can deploy capital. We now develop a measure of relative investment capacity. This measure shows at what extent one can deploy capital to a given portfolio without any constraints.

To build a measure of relative investment capacity of an asset *i* we first define investment capacity ratio (ICR) as the asset *i*'s weight in the market value weighted portfolio divided by the asset *i*'s weight in the portfolio *j*. This can be written as:

$$Investment\ capacity\ ratio_i^j = ICR_i^j = \frac{w_i^{MVW}}{w_i^j}$$

This ratio measures the investment capacity of an asset *i* as a percentage relative to the market value weighted portfolio. The intuition is that if this ratio is above 1 then the market has a high capacity to absorb capital into an asset *i* in a portfolio *j*. But if this ratio is below 1 then investors at some point would face capacity constraints with respect to asset *i* when deploying more and more capital into a portfolio *j*.

ICR is already a measure of investment capacity, but ICR's across portfolios are not comparable. Some portfolios use all available assets in the market portfolio and some use only a fraction of available assets. We therefore compute the $SIZE_j$ of the portfolio *j* relative to the market value weighted portfolio as the sum of market value weights of the assets in portfolio *j*. $SIZE_j$ takes the stocks in portfolio *j* and answers what percentage these stocks make of the market value weighted. Relative investment capacity (RIC) of a portfolio *j* is then the investment capacity ratio (ICR) multiplied by the portfolio SIZE as

$$RIC_j = ICR_j \times SIZE_j$$

Previous formula shows that to compare ICR's one need to normalize them by the SIZE of the portfolio. Relative investment capacities (RIC) are then comparable across portfolios.

We compute three different investment capacity ratios. The logical starting point is the bottleneck investment capacity ratio which is computed as the minimum ICR of a portfolio *j*

$$ICR_j^B = \min_i \left(\frac{w_i^{MVW}}{w_i^j} \right) \quad \text{where} \quad \frac{w_i^{MVW}}{w_i^j} \leq 1 \quad \text{and} \quad w_i^j > 0$$

Bottleneck ICR answers precisely how much money one can deploy to an alternative approach as a percentage of the market-value-weighted portfolio. This measure is the percentage of the portfolio *j*'s bottleneck weight relative to the market-value-weighted portfolio.

Since the bottleneck RIC can be prone to outliers we compute measures of RIC that focus on other than the bottleneck asset. One way is to compute a 5th percentile RIC, $RIC_j^{5\%}$, which is perhaps less prone to be an outlier. $RIC_j^{5\%}$ is computed as the lowest 5th percentile investment capacity ratio with the same constraints as the bottleneck. We also compute a relative investment capacity as a weighted average of investment capacity ratio to ensure our measure of investment capacity is robust. The weighted average investment capacity ratio is written as

$$ICR_j^{WA} = \frac{\sum_{i=1}^{n} w_i^{MVW} \left(\frac{w_i^{MVW}}{w_i^j} \right)}{SIZE_j} \quad \text{where} \quad \frac{w_i^{MVW}}{w_i^j} \leq 1 \quad \text{and} \quad w_i^j > 0$$

This measure has the following logic. RIC_j^{WA} is equal to 1 for the market value weighted portfolio because one can invest in the market value weighted portfolio at its full capacity. A portfolio that has an investment capacity ratio below or equal to one may have some investment constraints. We compute a market value weighted average of investment capacity ratios below one. This measure takes the size of the portfolio *j* relative to the market value weights into consideration.

The following examples further elaborate the logic behind RIC.

Example 1: The market-value-weighted portfolio always has the highest investment capacity of any portfolio, since it is the sum of all market values. $RIC^{MVW} = 1 \times 100$ =100 percent. Any other portfolio has a relative investment capacity below that of the market-value-weighted portfolio. In theory, the market-value-weighted index should have zero turn-over, but since market-value-weighted indices do not always utilise the full investment universe, the constituents in these indices change. A change in constituents would cause some trading in a market-value-weighted index.

Example 2: An equally-weighted portfolio has the following relative investment capacity: $RIC^{EW} = \frac{w_{smallest\ firm}^{MVW}}{w^{EW}} \times 1$. Intuitively, the bottleneck asset for the EW portfolio is the smallest company with the lowest market capitalisation. If a large investor were to deploy capital to an EW portfolio, this investor would have to stop deploying after having bought the entire stock of the firm with the smallest market capitalisation.

Example 3: Suppose we have two portfolios that have the same, 0.5, average investment capacity ratio below or equal to 1. Both portfolios use a subset of assets. Portfolio A uses 50% of the market value weighted capitalization and the portfolio B uses only 10%. In this case $RIC_A^{WA} = 0.5 \times 0.5 = 0.25$

and $RIC_B^{WA} = 0.1 \times 0.5 = 0.05$. This example shows that because portfolio A uses 50% of the market capitalization available it has much higher investment capacity than portfolio B.

Data and methodology

Our global dataset is the FTSE All-World Index Series which spans a time period from 1999 until 2011.[6] The FTSE All-World Index Series consists of large- and mid-cap stocks and covers about 90 to 95 percent of the market capitalisation of the FTSE Global Equity Index Series. The dataset excludes small-cap and emerging market stocks and may thus increase the overall investment capacity of the portfolio approaches we consider. We also apply the portfolio construction methodologies to the set of 30 Fama and French US industry portfolios available from Kenneth French's website. These portfolios use CRSP and COMPUSTAT data for US only and provide a robustness check with a longer time series spanning from 1975 until 2011.

To construct the portfolios, we use market value weights, fundamental weights, equal weights and the equal-risk-budget, equal-risk-contribution, minimum-variance and most-diversified portfolio approaches. These approaches are applied using monthly stock returns at individual stock, industry and country levels, where industry and country portfolios are market-value-weighted. We also consider GDP weights as a measure of economic size and apply these for country allocation.[7]

Each year, portfolios are formed and rebalanced at the end of December using available data over a five-year window. Our fundamental value is a five-year average of the book value of equity for each stock. Similarly, GDP is computed as a five-year average, and the equal-risk-budget portfolio is based on a five-year estimate of return volatility. When forming equal-risk-contribution, most-diversified and minimum-variance portfolios, one needs to estimate the variance-covariance matrix which is used in the optimisation. Consistent with heuristic weights, we use a five-year backward-looking window to estimate the matrix.

Estimating the variance-covariance matrix may not be straightforward. In principle, one would need a forward-looking variance-covariance matrix, and having a backward-looking estimate may not serve the purpose. Jumps caused by rare in-sample events unlikely to happen in the future can cause volatilities and correlations to exhibit shrinkage. Empirical research such as Jobson and Korkie (1980) and Pafka, Potters and Kondor (2004) has documented issues when estimating a variance-covariance matrix using historical realised returns. Ledoit and Wolf (2004) and Disatnik and Benninga (2007) argue that, when the number of stocks in the variance-covariance matrix is much larger than the number of historical observations covered in the matrix, the matrix will not be invertible. Even though the number of stocks would be smaller than the number of observations and the matrix is invertible, the matrix will still typically be ill-conditioned. Michaud (1989) argues that an ill-conditioned matrix is prone to estimation error and might contain extreme values, which would lead the portfolio optimisation algorithm to make extreme choices based on erroneous information.

There may be many ways to estimate a variance-covariance matrix. One could cluster the variance-covariance matrix according to risk factors, but this approach could bias portfolio weights towards these risk factors by construction. We chose to estimate the variance-covariance matrix by clustering it according to industry classification. We believe this is a neutral way to construct a variance-covariance matrix and it should not lead into portfolios that are tilted towards risk factors by construction. In practice, we estimate volatilities for each stock and correlation for each industry. We then assign the volatility estimates for each stock but use the industry correlation to compute covariance across stocks.

To understand what drives the portfolios based on alternative construction approaches, we analyse their exposure to some of the most well-known risk factors, such as size, value, momentum, reversal and the low-risk anomaly. We construct these factor portfolios using the same FTSE World dataset as for the portfolio construction. We do this by sorting the stock universe according to the relevant metric and forming equally-weighted decile portfolios. The return on each factor is then calculated by going long/short the top/bottom three decile portfolios on an equally-weighted basis.

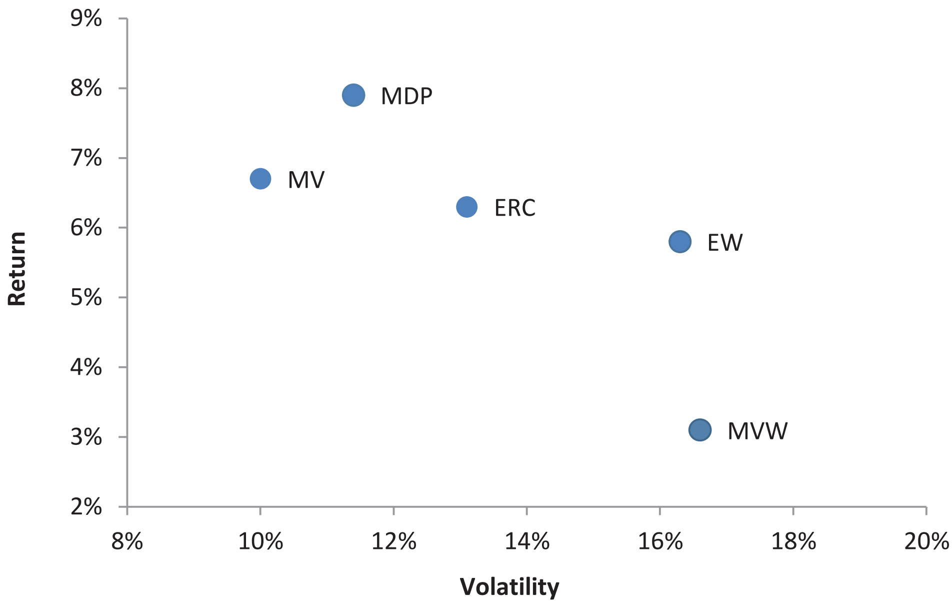
6 Since the FTSE data does not consist of small firms we ignore the small firm effect in our analysis. This is consistent with the Fama and French (1998) test of the value effect using international data.

7 We use GDP data from the US Department of Agriculture.

Empirical Results

Before discussing our empirical results we first review relevant findings related to our research. Choueifaty, Froidure and Reynier (2011) compare the most-diversified portfolio to a set of other portfolio construction approaches using daily MSCI World data spanning the period from 1999 to 2010. The authors construct their portfolios with a long-only constraint, semi-annual rebalancing and a one-year window for the covariance matrices. Their results suggest that all portfolios outperform the market-value-weighted index over the sample period considered. This outperformance is found even after subtracting transaction costs, and the most-diversified portfolio is found to have both the highest diversification ratio and the highest Sharpe ratio (see Chart 1).

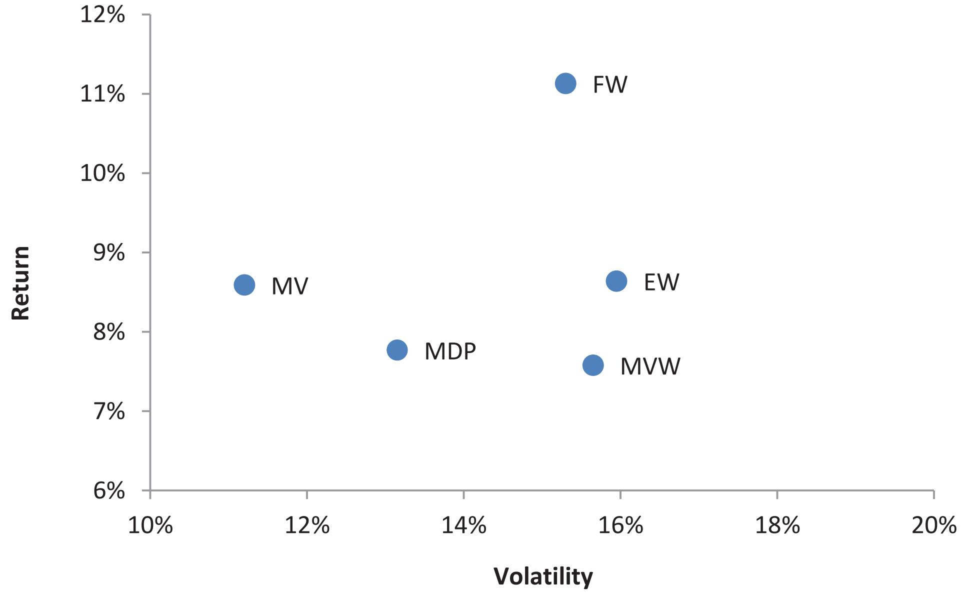
Chart 1: Annualised return vs. volatility plot (MSCI World data, 1999-2010)



Source: TOBAM, MSCI

Chow, Hsu, Kaleshnik and Little (2011) assess a set of portfolio construction approaches including fundamental weighting, equal weighting, minimum variance and the most-diversified portfolio on a global dataset from 1987 to 2009. They use stock return data from Datastream and financial accounting data from the Worldscope database. Consistent with Choueifaty, Froidure and Reynier (2011), they find that all alternative approaches outperform the market-value-weighted index. However, the fundamentally-weighted portfolio is found to have the highest Sharpe ratio over the entire sample period considered (see Chart 2).

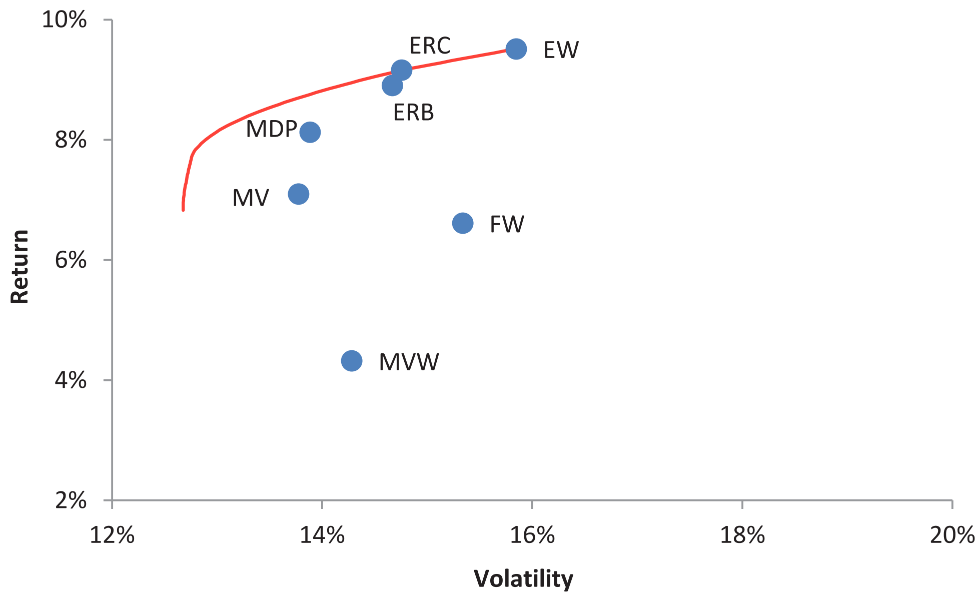
Chart 2: Annualised return vs. volatility plot (global data, 1987-2009)



Source: RAFI, Datastream, Worldscope

Charts 3-6 display results from our empirical study where annualised returns and volatilities are plotted for market value weights, fundamental weights, equal weights, minimum variance, most diversified, equal risk budget and equal risk contribution for all but country portfolios where we also plot results for GDP weights. Chart 3 is based on US individual stock return data. The results largely confirm the argument that market value weights underperform all of the other portfolio approaches. Equal risk budget, equal risk contribution, most diversified and equal weights provide the most attractive return-to-risk profiles, while all alternative approaches still provide a higher average return than market value weights. Consistent with the findings of Bloomfield, Leftwich and Long (1977) and DeMiguel, Garlappi and Uppal (2007), we find that the equally-weighted portfolio outperforms all of the other heuristic approaches and even some of the optimisation-based approaches in terms of Sharpe ratio.

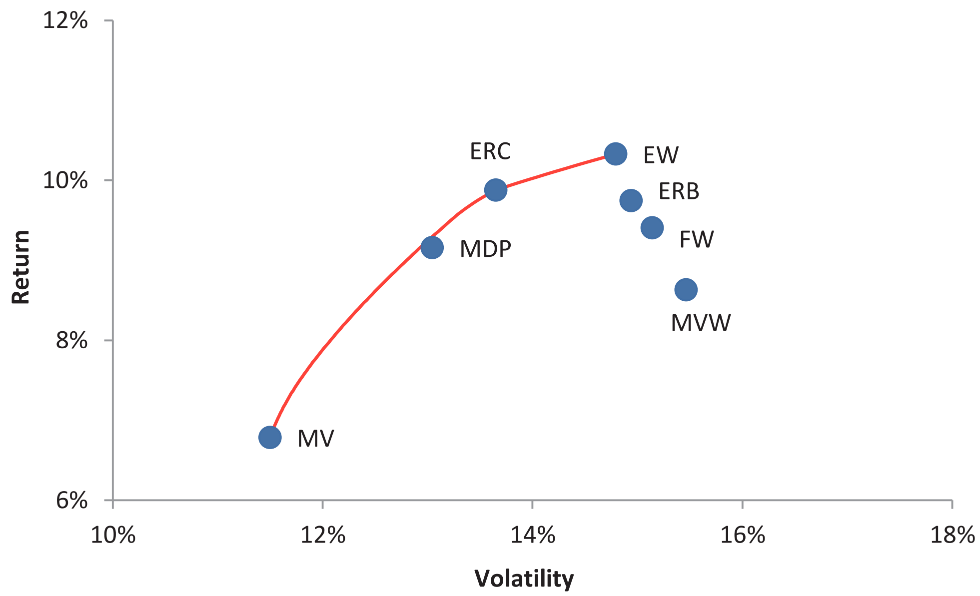
Chart 3: Annualised return vs. volatility plot (FTSE US stock-level data, 1999-2011)



Source: NBIM, FTSE

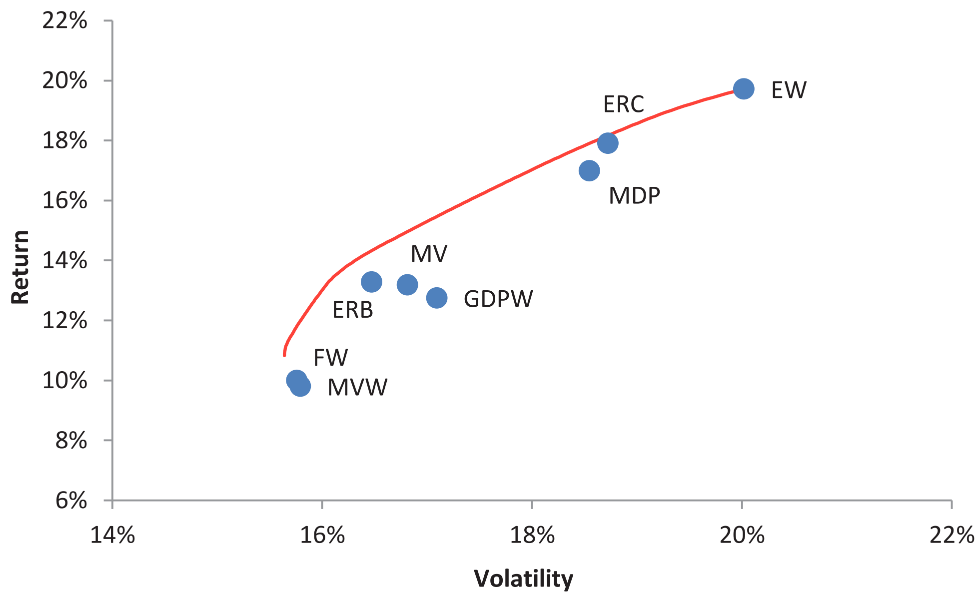
Chart 4 shows similar results for global industry portfolios – alternative portfolios outperform the market-value-weighted portfolio. While average returns generally are more evenly distributed among the industry portfolios, minimum variance still provides a significant reduction in portfolio volatility. In addition, equal risk contribution, most diversified and equal weights offer relatively attractive return-to-risk characteristics. However, we want to stress that the returns in Chart 4 and 5 are in USD and unhedged. Disentangling the effects from local equity returns and changes in FX rates is not possible.

Chart 4: Annualised return vs. volatility plot (FTSE global industry-level data, 1999-2011)



Source: NBIM, FTSE

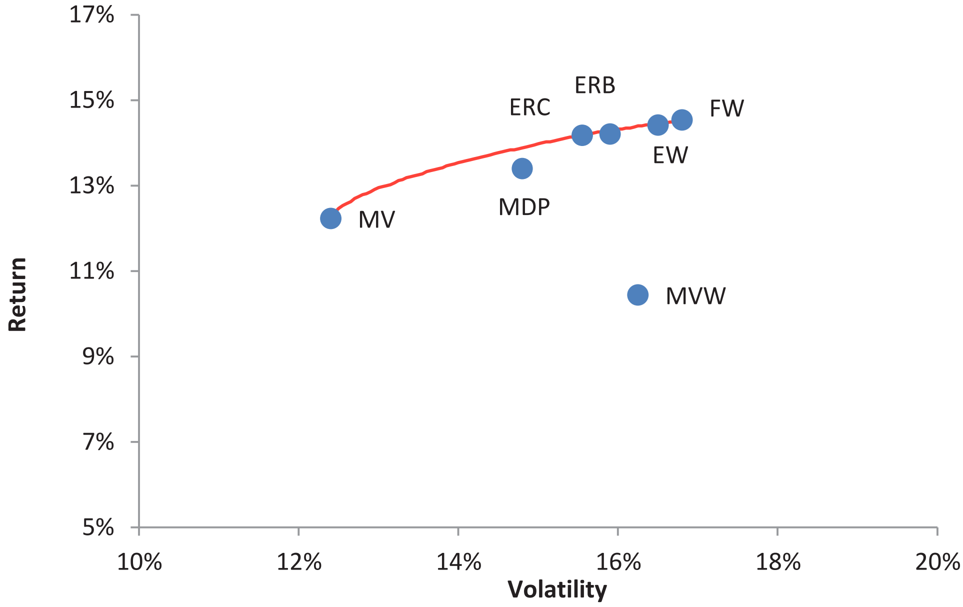
Chart 5: Annualised return vs. volatility plot (FTSE global country-level data, 1999-2011)



Source: NBIM, FTSE

Chart 6 depicts the results from running the same exercise on the 30 Fama and French US industry portfolios available from Kenneth French's website. These portfolios are specific to the US and provide a robustness check with a longer time series. The results are similar to those in Charts 3-5 where all alternative portfolio construction approaches outperform the market-value-weighted index. Here the fundamental weights are computed by using book-to-market instead of book value of equity. This provides much more attractive risk-return characteristics in the longer data sample.

Chart 6: Annualised return vs. volatility plot (Fama and French data, 1975-2011)



Source: NBIM, Kenneth R. French Data Library

Exposure to risk factors

Financial markets are characterised by anomalies that cannot be explained by conventional academic wisdom. The most well-known anomalies are: size, value, momentum, reversal and the low-risk anomaly.[8] There is evidence that small firms have outperformed large firms, at least in the past. However, we do not consider the small firm effect due to the fact that our data consists of mid and large cap stocks. Companies that are cheap with respect to some value metric (book-to-market, earnings-to-price and dividend yield) generate a high return on average. Since a strategy to buy one-year winners and sell one-year losers is profitable, stocks exhibit medium-term momentum. A short-term reversal strategy is to buy one-month losers and sell one-month winners. The low-risk anomaly refers to the observation that low-volatility stocks tend to outperform high-volatility stocks, exactly the opposite of what the basic intuition that higher risk is rewarded with a higher expected return tells us.

Past literature has shown that many alternative portfolios outperform the market-value-weighted portfolio. To really understand what drives these portfolios, we analyse them by keeping the anomalies (or risk factors) in mind. We estimate the factor loadings of each portfolio and measure the alphas with respect to the excess return on the market cap index and a combination of the factor portfolios.

Motivated by the same question, De Carvalho, Lu and Moulin (2011) study five risk-based approaches which they argue can be explained by market index, value, size, low beta and low residual volatility. The authors find that the most-diversified portfolio loads on low-beta stocks, while minimum variance loads on both low-beta stocks and low residual volatility. Equal risk budget and equal risk contribution

8 See, fro example, Banz (1981, Fama and French (1992), Fama and French (1993), Jegadeesh and Titman (993), Jegadeesh (1990) and Asness, Frazzini and Pedersen (2011).

are also found to load on low-beta stocks, but similarly to EW, they are also found to load on small-cap stocks. Chow, Hsu, Kaleshnik and Little (2011) and Choueifaty, Froidure and Reynier (2011) run similar factor regressions on a set of alternative portfolio approaches. Their main findings can be found in Table 2, which reveals that most of the portfolios can be explained to a great extent by exposures to the market portfolio, value and size factors, while the momentum factor seems to contribute very little.

Table 2: Exposure to risk factors (Research Affiliates and TOBAM data)

	Table: 1987-2011					
	Annual Alpha	Market (Mktr-Rf)	Small Cap (SMB)	Value (HML)	Momentum (MOM)	R^2
FW: Fundamentally Weighted	2.18 %	0.97	0.04	0.33	-0.09	0.97
EW: Equally Weighted	0.77 %	1.02	0.26	0.03	-0.01	0.98
MV: Minimum Variance	1.25 %	0.63	0.00	0.14	-0.01	0.73
ERC: Equal Risk Contribution*	0.14 %	0.96	0.41	0.06	NaN	0.93
MDP: Most Diversified Portfolio*	2.26 %	0.57	0.31	0.16	NaN	0.80

This table shows non-market-cap index returns relative to the market portfolio as well as the small-cap, value and momentum factors (when available). We show results from two sources: Research Affiliates and TOBAM. These datasets may differ and they also use different time periods. The RAFI data cover 1987 until 2009. The TOBAM data cover 1999 until 2010. Statistically significant coefficients are highlighted.
Source: Research Affiliates (1987-2009)
**MDP and ERC are from TOBAM and cover the period from 1999 to 2010.*

We estimate regressions similar in spirit to De Carvalho, Lu and Moulin (2011), Chow, Hsu, Kaleshnik and Little (2011) and Choueifaty, Froidure and Reynier (2011) using the data as described in the data section. The results are shown in Tables 3 and 4. In addition to the market portfolio, we consider value and volatility factors. MV attempts to minimize portfolio variance and MDP aims to find a most diversified portfolio. The fact that these approaches have low market betas (0.84 and 0.81 respectively) is consistent with their goal to reduce risk. In addition MV has a high slope on a low risk factor indicating this portfolio is concentrated towards low-risk stocks.

FW loads on value with a positive beta of 0.22 on our version of the HML factor. All other approaches than MDP and MV have positive value betas. ERB and MV have the highest loadings on the volatility factor, which is intuitive because ERB is based on equalising volatility budget and MV minimises volatility risk. Table 4 shows results for the industry portfolios. These results are consistent with Table 3 and show that alternative portfolio approaches provide consistent exposure even at the industry portfolio level. We do not report these results for the country level portfolios because there is very little variation in the effects when using aggregate market portfolios.

Table 3: Exposure to risk factors, stock-level data

Table: 1999-2011					
	Annual Alpha	Market (Mktr-Rf)	Value (HML)	Volatility (VOL)	R^2
EW: Equally Weighted	3.33 %	0.98	0.25	0.01	0.92
ERB: Equal Risk Budget	2.91 %	0.96	0.27	0.08	0.92
ERC: Equal Risk Contribution	3.27 %	0.95	0.23	0.05	0.92
MDP: Most Diversified Portfolio	4.00 %	0.81	-0.03	-0.03	0.74
MV: Minimum Variance	3.02 %	0.84	0.05	0.18	0.53
FW: Fundamentally Weighted	0.87 %	1.03	0.22	0.06	0.97

This table shows various portfolio exposures to style factors. Portfolios are FW, EW, ERB, ERC, MDP and MV. The factors used are market index, small cap, value, momentum and low volatility. Statistically significant coefficients are highlighted.
Source: NBIM calculations, FTSE

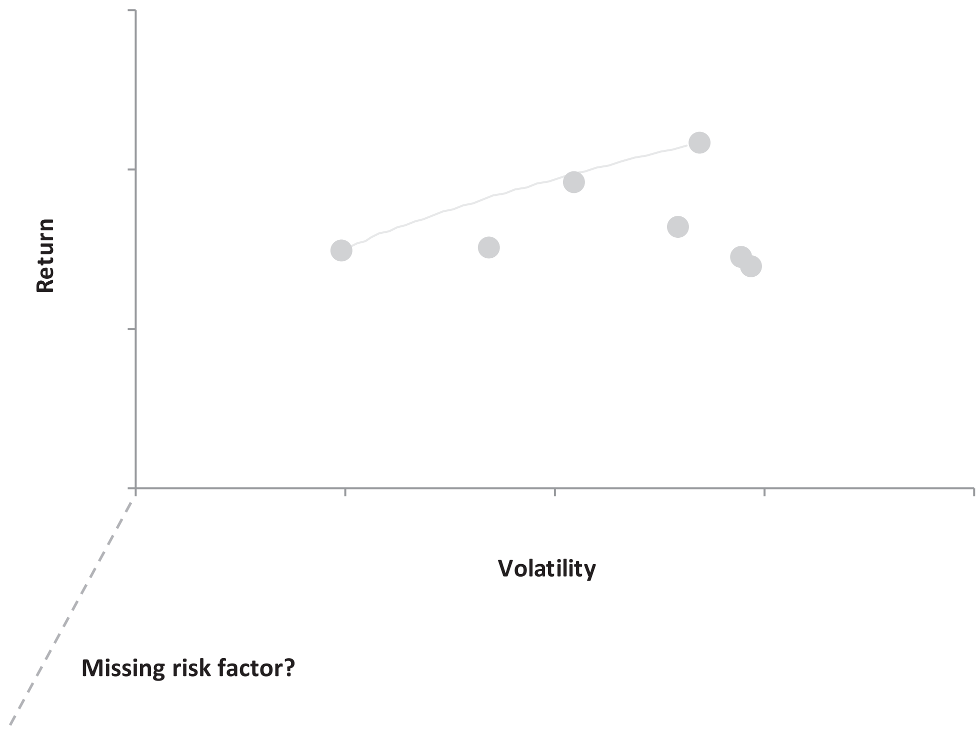
Table 4: Exposure to risk factors (industry-level data)

Table: 1999-2011					
	Annual Alpha	Market (Mktr-Rf)	Value (HML)	Volatility (VOL)	R^2
EW: Equally Weighted	1.34 %	0.99	0.04	0.06	0.98
ERB: Equal Risk Budget	-0.33 %	0.99	0.19	0.09	0.97
ERC: Equal Risk Contribution	0.88 %	0.94	0.09	0.11	0.96
MDP: Most Diversified Portfolio	1.57 %	0.88	-0.04	0.11	0.88
MV: Minimum Variance	-1.58 %	0.78	0.16	0.20	0.79
FW: Fundamentally Weighted	-0.44 %	1.01	0.15	0.08	0.99

This table shows various portfolio exposures to style factors. Portfolios are FW, EW, ERB, ERC, MDP and MV. The factors used are market index, small cap, value, momentum and low volatility. Statistically significant coefficients are highlighted.
Source: NBIM calculations, FTSE

An important point to highlight when operating in a return to standard deviation space is that a portfolio that moves towards the upper left corner (higher return, lower standard deviation) may be taking on some omitted priced risk factor (see Figure 7). Fama and French (1993) argued that the value effect is based on a risk not captured by the market portfolio. The authors argue that size and book to market proxy for distress risk, and market participants require a risk premium to bear this risk. It is more difficult to explain why momentum and the low-risk anomaly would be risk factors per se, but they might capture some common variation around the business cycle. If these factors represent a compensation for risk not explained by the CAPM, then the mean-variance framework would not be a sufficient framework for portfolio optimisation. One would have to consider and understand these risks beyond standard risk measures such as beta and volatility.

Figure 7: Illustration of how return to standard deviation space can be expanded



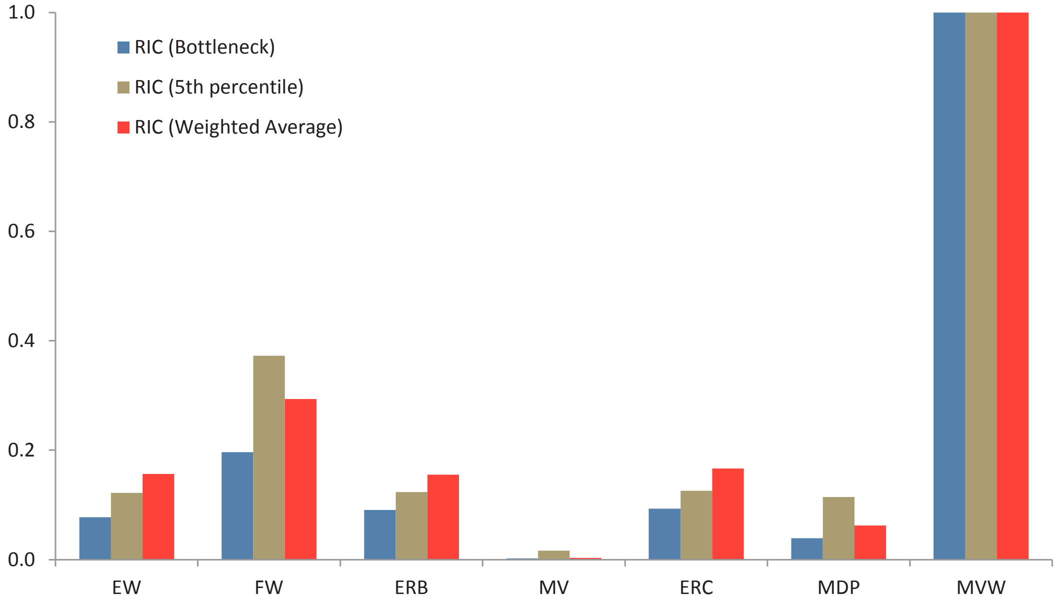
Relative investment capacity

We have shown that alternative portfolios have appealing return-to-risk characteristics. If a large investor or the market as a whole were to allocate money to an alternative portfolio, it is important to know how much one can deploy capital to a specific portfolio. We introduced a novel measure of relative investment capacity (RIC) to answer this question.

We now calculate three versions of RIC for all portfolio approaches at stock, industry, and country level. Three versions are the bottleneck, 5th percentile, and weighted average. The results are depicted in Charts 8-10 and more detailed figures are shown in Tables 5-7. These tables break the RIC into investment capacity ratios and portfolio size. Chart 8 shows that FW has the highest RIC of all the stock-level portfolios. Chart 9 shows that FW and ERB have very high investment capacity among industry portfolios. Chart 10 shows country level RIC. FW again stands out with a high RIC and the most disappointing RIC is in the GDPW.

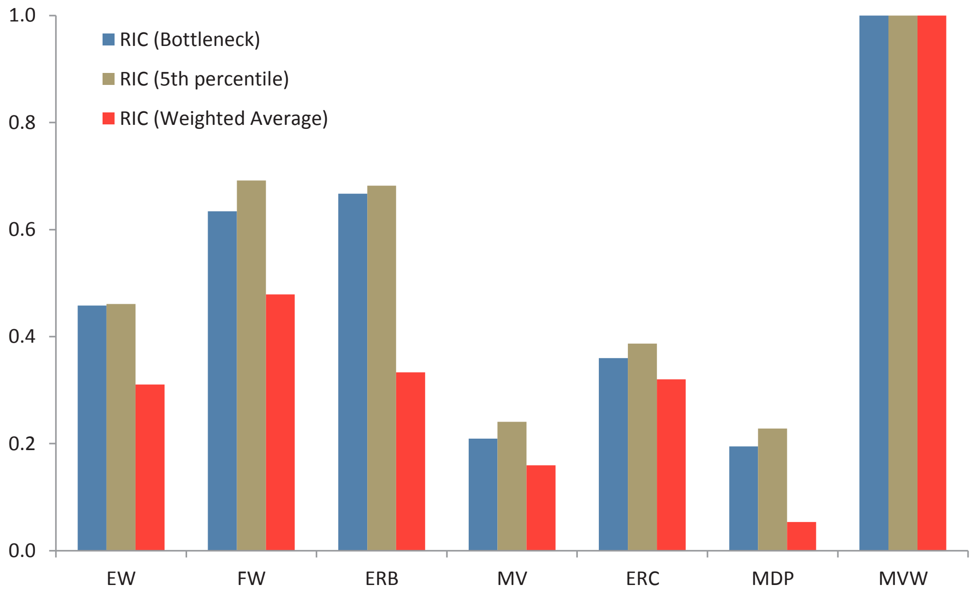
Approaches based on optimization have very low RICs. This makes us think whether these approaches are suitable as benchmark portfolios. If one were to consider low RIC strategies as an active investment strategy is, of course, a different story.

Chart 8: Relative investment capacity measures (FTSE US stock-level data, 1999-2011)



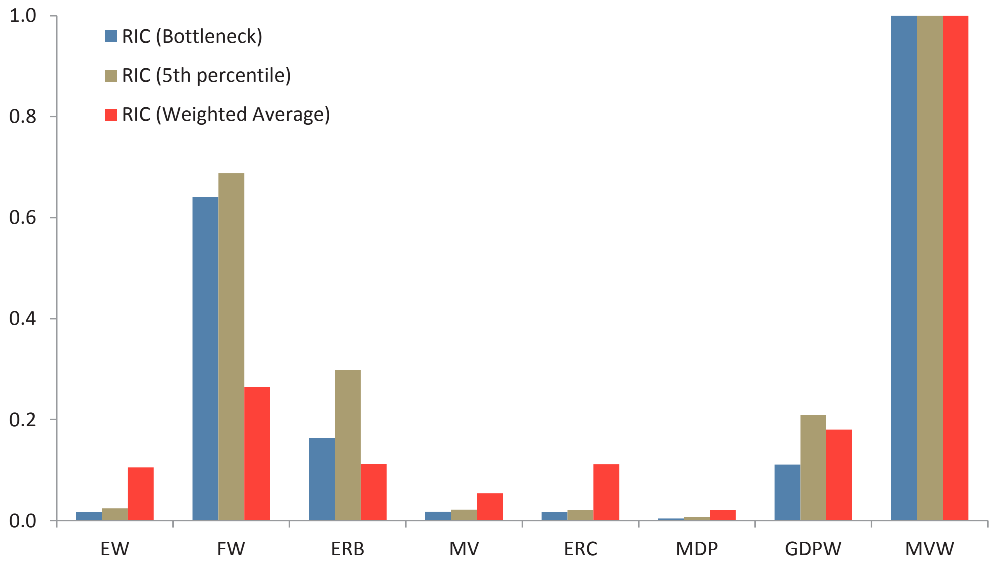
Source: NBIM calculations, FTSE

Chart 9: Relative investment capacity measures (FTSE World industry-level data, 1999-2011)



Source: NBIM calculations, FTSE

Chart 10: Relative investment capacity measures (FTSE World country-level data, 1999-2011)



Source: NBIM calculations, FTSE

Table 5: Relative investment capacity, size and turnover (FTSE US stock-level data, 1999-2011)

	EW	FW	ERB	MV	ERC	MDP	MVW
ICR (Bottleneck)	0.08	0.20	0.09	0.13	0.09	0.04	1.00
ICR (5th percentile)	0.12	0.37	0.12	1.00	0.13	0.11	1.00
ICR (Weighted Average)	0.16	0.29	0.16	0.19	0.17	0.06	1.00
Portfolio SIZE	100.0 %	100.0 %	100.0 %	1.6 %	100.0 %	100.0 %	100.0 %
RIC (Bottleneck)	0.08	0.20	0.09	0.00	0.09	0.04	1.00
RIC (5th percentile)	0.12	0.37	0.12	0.02	0.13	0.11	1.00
RIC (Weighted Average)	0.16	0.29	0.16	0.00	0.17	0.06	1.00
Turnover	19.9 %	16.2 %	18.8 %	88.3 %	19.1 %	41.7 %	12.7 %

Source: NBIM calculations, FTSE

Table 6: Relative investment capacity, size and turnover (FTSE World industry-level data, 1999-2011)

	EW	FW	ERB	MV	ERC	MDP	MVW
ICR (Bottleneck)	0.46	0.63	0.67	0.21	0.36	0.20	1.00
ICR (5th percentile)	0.46	0.69	0.68	0.24	0.39	0.23	1.00
ICR (Weighted Average)	0.31	0.48	0.33	0.16	0.32	0.05	1.00
Portfolio SIZE	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %	100.0 %
RIC (Bottleneck)	0.46	0.63	0.67	0.21	0.36	0.20	1.00
RIC (5th percentile)	0.46	0.69	0.68	0.24	0.39	0.23	1.00
RIC (Weighted Average)	0.31	0.48	0.33	0.16	0.32	0.05	1.00
Turnover	9.0 %	8.8 %	9.1 %	18.1 %	9.2 %	14.1 %	8.5 %

Source: NBIM calculations, FTSE

Table 7: Relative investment capacity, size and turnover (FTSE World country-level data, 1999-2011)

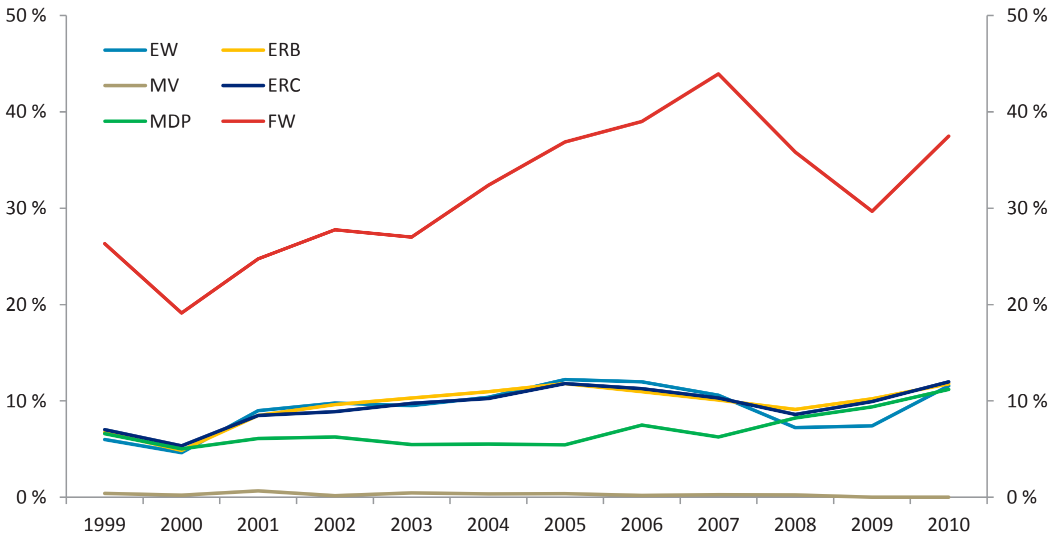
	EW	FW	ERB	MV	ERC	MDP	GDPW	MVW
ICR (Bottleneck)	0.02	0.64	0.16	0.02	0.02	0.00	0.11	1.00
ICR (5th percentile)	0.02	0.69	0.30	0.02	0.02	0.01	0.21	1.00
ICR (Weighted Average)	0.11	0.26	0.11	0.06	0.11	0.02	0.18	1.00
Portfolio SIZE	100.0 %	100.0 %	100.0 %	90.5 %	100.0 %	100.0 %	100.0 %	100.0 %
RIC (Bottleneck)	0.02	0.64	0.16	0.02	0.02	0.00	0.11	1.00
RIC (5th percentile)	0.02	0.69	0.30	0.02	0.02	0.01	0.21	1.00
RIC (Weighted Average)	0.11	0.26	0.11	0.05	0.11	0.02	0.18	1.00
Turnover	14.6 %	10.6 %	12.1 %	55.5 %	15.6 %	37.2 %	11.2 %	9.9 %

Source: NBIM calculations, FTSE

However, the historical 5th percentile RICs displayed in Charts 11-13 show that the RIC of FW has been high and increasing over time. MDP and MV typically have had the lowest RICs over the sample period, again at stock, industry and country level.

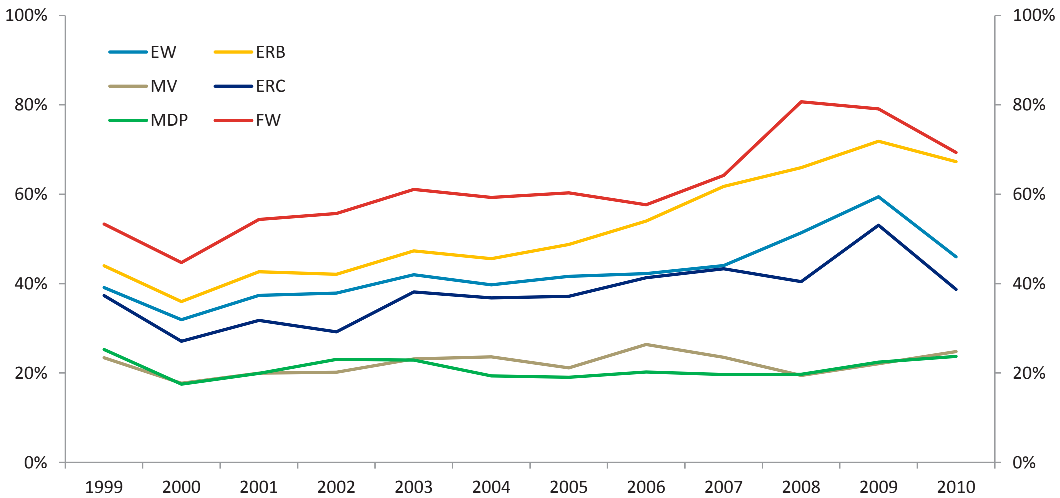
Similarly, MV and MDP have higher turnover than the other approaches, which generally have turnover levels that are more comparable to the market-value-weighted index. This is particularly strong in the stock-level portfolios. In general, the main difference between the results found from looking at the stock-level portfolios versus the industry portfolios is in the overall level of RIC and turnover. Since the industry portfolios are market-value-weighted, any stock-specific effects have been faded before applying the alternative portfolio construction approaches.

Chart 11: Historical 5th percentile relative investment capacity (FTSE US stock-level data, 1999-2011)



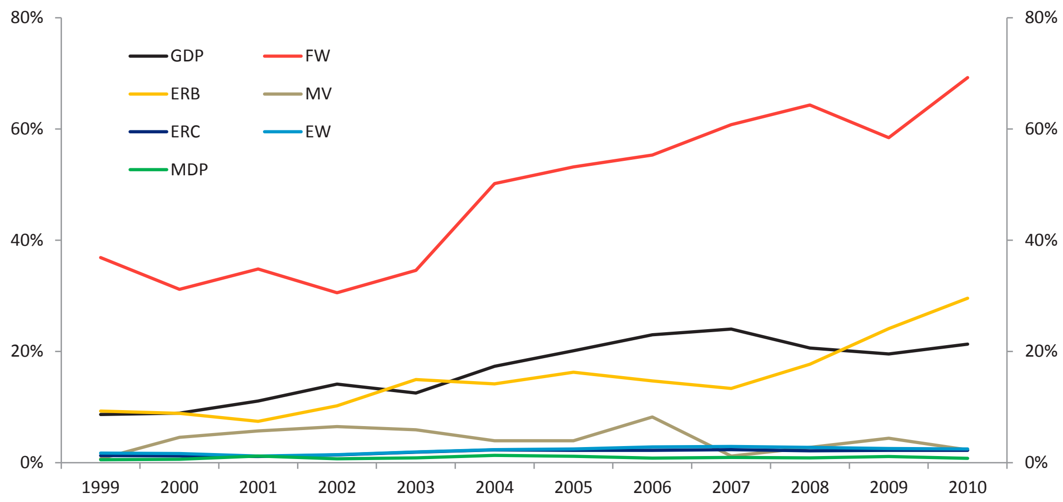
Source: NBIM calculations, FTSE

Chart 12: Historical 5th percentile relative investment capacity (FTSE World industry-level data, 1999-2011)



Source: NBIM calculations, FTSE

Chart 13: Historical 5th percentile relative investment capacity (FTSE World country-level data, 1999-2011)



Source: NBIM calculations, FTSE

Conclusions

Alternatives to the market-value-weighted portfolio have appealing Sharpe ratios and provide different ways to participate in the value and low-risk anomalies. Alternative portfolios have an investment capacity lower than that of the market-value-weighted portfolio. High investment capacity and liquidity may be a desirable property for an investor, but it comes at a cost – lower expected return.

Deviating away from market cap weights the portfolio by construction becomes more tilted towards smaller value stocks. At the same time, investment capacity and liquidity become lower as well. For this to make sense from an investor's point of view, lower capacity and liquidity should provide higher returns over time. Some of the financial market anomalies may be illiquid and beyond the investment scope of large investors.

We have shown that the measure of relative investment capacity can be a useful tool to understand the investability of different portfolios. Our RIC measures suggest that FW portfolio has the highest capacity at stock, industry, and country level. ERB portfolio has very high capacity at industry level. GDP weights have disappointingly low investment capacity especially compared to FW.

If one were to invest in alternative portfolios important questions still remain. Weighting equities directly using a fundamental size of the firm would ignore capital structure and float (percentage of the company being traded).

Our results for relative investment capacity should be of interest to investors considering alternative portfolio weights either for investments or for investment policy benchmarks. We advocate the idea that investment policy benchmarks should have high and stable relative investment capacity.

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